

Synthesis of Transformer Coupled Multiple Frequency Circulators with Chebyshev Characteristics

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Abstract—This paper presents a theory for broad-band matching of stripline junction circulators for operation in two or more frequency bands. In this theory it is assumed that the matching network is composed of cascaded transmission line transformers each of which is an odd multiple of a quarter-wavelength at the center frequencies. The conditions for simultaneous Chebyshev response in multiple frequency bands are determined, and it is investigated to what extent these conditions can be satisfied by stripline circulator junctions. Thus by using a first-order theory, it is shown that if a circulator junction, adjusted for double frequency operation, is matched for Chebyshev response by a transformer of proper length around one of the circulation frequencies then it is also matched for Chebyshev response around the other circulation frequency, provided that the same operation mode is used above and below material resonance. A routine for broad-band multiple frequency matching is proposed for junctions where Chebyshev response is not obtainable. Finally the properties of some externally matched circulators designed according to the theories are shown.

I. INTRODUCTION

THE POSSIBILITY of broad-band matching of stripline junction circulators for single frequency band operation by using quarter-wave transformers has been known for many years. In 1972 Helszajn presented a paper giving the theoretical basis for the synthesis of quarter-wave coupled stripline circulators with Chebyshev characteristics [1].

In the early 1960's it was found experimentally that circulator junctions can be adjusted for operation at two separate frequency bands [2], [3]. These circulators were operated at the (1,1)-mode below and above the ferromagnetic resonance frequency. In recent years the possibility of using multiple resonance modes to obtain broad-band circulator operation [4], [5] as well as diplexer function [6], [7] has been demonstrated. A first-order theory for the design of stripline circulators for multiple frequency operation has been developed based on the simplification of taking into account only the dominant resonance mode at each circulation frequency [8], [9]. It was shown that a multitude of circulators for double and multiple frequency band opera-

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tion can be constructed by using combinations of only some of the lower order modes. Like conventional stripline circulators they are narrow-banded if they are directly coupled to an external network. As indicated by the loaded Q -factors related to the different operation modes the bandwidths can be increased by using external matching networks.

The advances in the field of stripline circulators for multiple frequency operation thus emphasize the necessity of studying networks for simultaneous matching within several frequency bands.

To get Chebyshev response in multiple frequency bands simultaneously by using transmission line transformers, we have found it necessary to further develop the theories given in [1]. In this paper we allow the transmission lines to have an electric length equal to an arbitrary, odd, number of quarter-wavelengths. Thereby it is possible to use the same matching network to get Chebyshev response in two or more frequency bands.

The theory is developed along the following lines.

1) Two networks with slightly different properties are presented. If the network parameters are chosen properly the input admittances of the networks approximate the complex gyrator admittance for a stripline circulator junction in some vicinity of a parallel resonance frequency.

2) It is shown that both these equivalent networks can be matched for Chebyshev response by the use of multi-quarter-wave transformers.

3) The conditions for simultaneous Chebyshev response around two or more resonance frequencies of the equivalent networks are derived.

4) Finally it is discussed how, and to what extent these conditions can be satisfied by proper adjustment of a stripline circulator junction.

II. TWO MODELS OF A CIRCULATOR JUNCTION PORT

In this work two different equivalent networks for circulator junctions have been used. If the parameters of the networks are chosen properly the input admittances of the networks approximate the admittance of a stripline circulator junction port in the vicinity of a parallel resonance frequency under the condition that the other ports are matched. This admittance is known as the complex gyrator

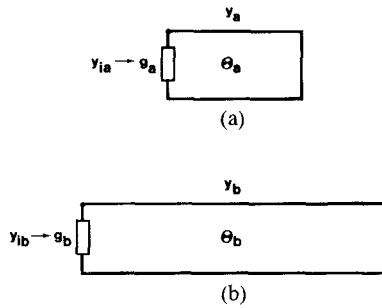


Fig. 1. Equivalent networks for a circulator junction;

g_a, g_b shunt conductances;
 y_a, y_b characteristic admittances of the transmission lines;
 θ_a, θ_b electric lengths of the transmission lines, $\theta_a = \pi f_c / 2f_c$ and $\theta_b = u\pi f_c / 2f_c$;
 y_{ia}, y_{ib} input admittances of the networks;
 f_c resonance frequency;
 u odd integer constant.

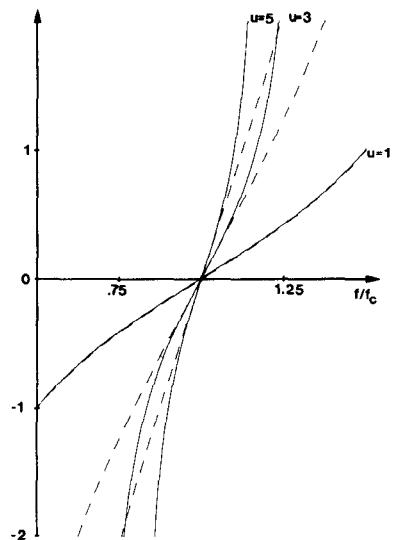


Fig. 2. Normalized input susceptance of the equivalent networks.

$$\text{Im}\{y_{ia}\}/y_a = -u \cot(\pi f_c / 2f_c) \quad (\cdots)$$

$$\text{Im}\{y_{ib}\}/y_b = -\cot(u\pi f_c / 2f_c) \quad (\text{---}).$$

admittance or the equivalent admittance of the junction. One of the networks is used by Helszajn [1] and consists of a quarter-wavelength short-circuited stub in parallel with a shunt conductance. The second equivalent network is similar to the first one but the length of the stub is an arbitrary, odd, multiple, u , of a quarter-wavelength at the given resonance frequency, see Fig. 1(a) and (b). The input admittances of the circuits are

$$y_{ia} = g_a - jy_a \cot \theta_a \quad (1)$$

$$y_{ib} = g_b - jy_b \cot \theta_b. \quad (2)$$

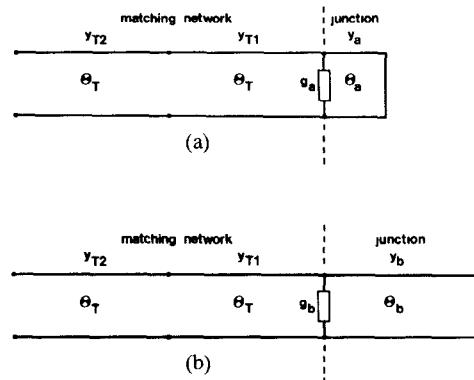
The conditions for similarity between the networks at resonance can be written

$$y_{ia}|_{f_c} = y_{ib}|_{f_c} \quad (3)$$

$$\frac{dy_{ia}}{df} \Big|_{f_c} = \frac{dy_{ib}}{df} \Big|_{f_c}. \quad (4)$$

To satisfy (3) and (4) we get

$$g_a = g_b \quad (5)$$

Fig. 3. Equivalent networks for a circulator junction with two matching transformers. y_{T1}, y_{T2} = characteristic admittances for the transformer lines. θ_T = electric length of the transformer lines, $\theta_a = u\pi f_c / 2f_c$.

and

$$y_a = uy_b. \quad (6)$$

A comparison of the susceptance of the two networks, with parameters according to (5) and (6), is made in Fig. 2. It is evident that the susceptance properties of the two networks differ substantially for frequencies at some distance from the resonance frequency if u is greater than one. Thus in the design of matched stripline circulators for multiple frequency operation, the equivalent network that best fits the complex gyrator admittance of the junction around the resonance frequency may be chosen. It should be observed that if these equivalent networks are to be applied at more than one resonance frequency then the network parameters will, in general, take different values at each resonance.

III. MATCHING FOR CHEBYSHEV RESPONSE USING MULTI-QUARTER-WAVE TRANSFORMERS

The two equivalent networks supplied with two matching transformers are shown in Fig. 3. The electrical lengths indicated in Fig. 3 are: $\theta_T = \theta_b = u\pi f_c / 2f_c$; $\theta_a = \pi f_c / 2f_c$. By using electrically long transformers it is possible to match junctions with high loaded Q -factors for Chebyshev response. The procedure of synthesizing the networks for Chebyshev response is analogous with the one presented by Helszajn for the special case $u=1$ [1]. Therefore, only the results will be listed. Specifying the bandwidth BW and the maximum VSWR, r , of the Chebyshev response we get with notations according to Fig. 3.

A. Circuit a, One Transformer

$$g_a = \frac{r - \sin^2(u\theta)}{r \cos^2(u\theta)} \quad (7)$$

$$y_a = \sqrt{\frac{g_a}{r}} (rg_a - 1) \tan(\theta) \sin(u\theta) \cos(u\theta) \quad (8)$$

$$y_{T1} = \sqrt{rg_a} \quad (9)$$

where (7) and (8) should be evaluated with

$$\theta = \cos^{-1} \left\{ \frac{1}{\sqrt{2}} \cos \left[\left(1 + \frac{BW}{2} \right) \cdot \frac{\pi}{2} \right] \right\}. \quad (10)$$

B. Circuit a, Two Transformers

The solution is given implicitly by the following set of relations:

$$y_{T2}^2 = \frac{(1+\sqrt{g_a})^2 \sin^2(u\theta) \cos^2(u\theta)}{1 - [\sqrt{g_a} \cos^2(u\theta) - \sin^2(u\theta)]^2} \quad (11)$$

$$y_{T1}^2 = g_a y_{T2}^2 \quad (12)$$

$$y_a = \frac{CD-AB}{B^2+D^2} \cdot \tan \theta \quad (13)$$

$$\begin{aligned} & \frac{r-1}{r+1} [(Dg_a + A + By_a \cot \theta)^2 + (C + Bg_a - Dy_a \cot \theta)^2]^{1/2} \\ & = [(Dg_a - A - By_a \cot \theta)^2 + (C - Bg_a - Dy_a \cot \theta)^2]^{1/2} \quad (14) \end{aligned}$$

where

$$A = \cos^2(u\theta) - \frac{y_{T1}}{y_{T2}} \sin^2(u\theta) \quad (15)$$

$$B = \left(\frac{1}{y_{T2}} + \frac{1}{y_{T1}} \right) \sin(u\theta) \cos(u\theta) \quad (16)$$

$$C = (y_{T2} + y_{T1}) \sin(u\theta) \cos(u\theta) \quad (17)$$

$$D = \cos^2(u\theta) - \frac{y_{T2}}{y_{T1}} \sin^2(u\theta). \quad (18)$$

Here (11) and (13) should be evaluated with

$$\theta = \cos^{-1} \left\{ \sqrt{\frac{3}{4}} \cos \left[\left(1 + \frac{BW}{2} \right) \cdot \frac{\pi}{2} \right] \right\} \quad (19)$$

while (14) with

$$\theta = \cos^{-1} \left\{ \frac{1}{2} \cos \left[\left(1 + \frac{BW}{2} \right) \cdot \frac{\pi}{2} \right] \right\}. \quad (20)$$

The values of θ according to (19) and (20) correspond to the upper reflection zero and reflection maximum, respectively, in the frequency band for Chebyshev response.

Circuit *b* can be considered as a generalization of Helszajn's corresponding network [1]. Due to the periodic properties of the distributed transmission lines all of them having the same electric length, a Chebyshev response obtained in a frequency band corresponding to $u=1$ will be repeated in higher frequency bands corresponding to $u=3, 5, 7, \dots$. Thus the relative bandwidth will be reduced by a corresponding factor. These statements are valid for the matching of the equivalent network, leaving the consequences of inserting the circulator junction to the discussion in Section V. The following synthesis procedure results.

C. Circuit b, One Transformer

$$g_b = \frac{r - \sin^2 \theta}{r \cos^2 \theta} \quad (21)$$

$$y_b = \sqrt{\frac{g_b}{r}} (rg_b - 1) \sin^2 \theta \quad (22)$$

$$y_{T1} = \sqrt{rg_b} \quad (23)$$

where (21) and (22) should be evaluated with

$$\theta = \cos^{-1} \left\{ \frac{1}{\sqrt{2}} \cos \left[\left(1 + \frac{u \cdot BW}{2} \right) \cdot \frac{\pi}{2} \right] \right\}. \quad (24)$$

D. Circuit b, Two Transformers

$$y_{T2}^2 = \frac{(1+\sqrt{g_b})^2 \sin \theta \cos^2 \theta}{1 - [\sqrt{g_b} \cos^2 \theta - \sin^2 \theta]^2} \quad (25)$$

$$y_{T1}^2 = g_b y_{T2}^2 \quad (26)$$

$$y_b = \frac{CD-AB}{B^2+D^2} \tan \theta \quad (27)$$

$$\begin{aligned} & \frac{r-1}{r+1} [(Dg_b + A + By_b \cot \theta)^2 + (C + Bg_b - Dy_b \cot \theta)^2]^{1/2} \\ & = [(Dg_b - A - By_b \cot \theta)^2 + (C - Bg_b - Dy_b \cot \theta)^2]^{1/2} \quad (28) \end{aligned}$$

where

$$A = \cos^2 \theta - \frac{y_{T1}}{y_{T2}} \sin^2 \theta \quad (29)$$

$$B = \left(\frac{1}{y_{T2}} + \frac{1}{y_{T1}} \right) \sin \theta \cos \theta \quad (30)$$

$$C = (y_{T2} + y_{T1}) \sin \theta \cos \theta \quad (31)$$

$$D = \cos^2 \theta - \frac{y_{T2}}{y_{T1}} \sin^2 \theta. \quad (32)$$

Here (25) and (27) should be evaluated with

$$\theta = \cos^{-1} \left\{ \sqrt{\frac{3}{4}} \cos \left[\left(1 + \frac{u \cdot BW}{2} \right) \cdot \frac{\pi}{2} \right] \right\} \quad (33)$$

while (28) with

$$\theta = \cos^{-1} \left\{ \frac{1}{2} \cos \left[\left(1 + \frac{u \cdot BW}{2} \right) \cdot \frac{\pi}{2} \right] \right\}. \quad (34)$$

The values of θ calculated in (24), (33), and (34), correspond to reflexion extrema in the lowest frequency band for Chebyshev response.

IV. CONDITIONS FOR CHEBYSHEV RESPONSE AT MULTIPLE FREQUENCY BANDS

With the synthesis expressions derived in Section III, it is evident that both the equivalent networks for circulator junctions can be matched for Chebyshev response around any resonance frequency, where u according to Fig. 3 is an odd integer. Consider now a circulator junction with matching network. We want the circuit to give Chebyshev response around two frequencies, f_1 and f_2 , such that $f_2/f_1 = u_2/u_1$, where u_1 and u_2 are odd integers. The maximum VSWR in the frequency bands around f_1 and f_2 where the circuit is to be matched is r_1 and r_2 , respectively. For simplicity we assume that the characteristic admittances of the matching transformers can be considered frequency independent. By anticipating the result of the analysis performed in Section V we also postulate that the shunt conductances of the equivalent networks are the same at f_1 and f_2 . The resulting equivalent networks at f_1 and f_2 with two matching transformers are shown in Fig. 4. The electric lengths of the transmission lines are: $\theta_{T1} = \theta_{b1} = u_1 \pi f/2f_1$; $\theta_{T2} = \theta_{b2} = u_2 \pi f/2f_2$; $\theta_{a1} = \pi f/2f_1$; $\theta_{a2} = \pi f/2f_2$. As $u_1/f_1 = u_2/f_2$ we find that $\theta_{T1}(f) \equiv \theta_{T2}(f)$ which should be the case since these angles represent the

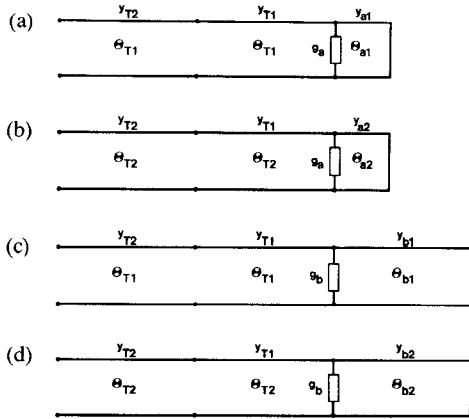


Fig. 4. Equivalent networks for a matched circulator junction with center frequencies f_1 and f_2 . (a) $f_c = f_1$. (b) $f_c = f_2$. (c) $f_c = f_1$. (d) $f_c = f_2$.

lengths of the same physical transmission lines. The fact that $\theta_{a1}(f) \neq \theta_{a2}(f)$ is no problem as this part of the network is a mathematical representation for the junction and is not realized in the form of transmission lines. Using the parameters defined in this figure the conditions for simultaneous Chebyshev response are found from the relations in Section III.

A. Circuit a, One Transformer

Equations (7) and (9) give the conditions

$$\frac{r_1 - \sin^2(u_1\theta_1)}{r_1 \cos^2(u_1\theta_1)} = \frac{r_2 - \sin^2(u_2\theta_2)}{r_2 \cos^2(u_2\theta_2)} \quad (35)$$

$$r_1 g_a = r_2 g_a. \quad (36)$$

From (36) we find

$$r_1 = r_2 \quad (37)$$

which inserted into (35) gives

$$\cos^2(u_1\theta_1) = \cos^2(u_2\theta_2). \quad (38)$$

The difference between the transformer lengths at f_2 and f_1 is $(u_2 - u_1) \cdot \pi/2$. Considering the reflexion zeros of the Chebyshev responses occurring above f_1 and f_2 as indicated by (10), the solution of (38) is given by

$$u_2\theta_2 = u_1\theta_1 + (u_2 - u_1) \cdot \frac{\pi}{2}. \quad (39)$$

From (10) we find the following relations for the bandwidths, $BW1$ and $BW2$, around f_1 and f_2 :

$$\theta_1 = \cos^{-1} \left\{ \frac{1}{\sqrt{2}} \cos \left[\left(1 + \frac{BW1}{2} \right) \cdot \frac{\pi}{2} \right] \right\} \quad (40)$$

$$\theta_2 = \cos^{-1} \left\{ \frac{1}{\sqrt{2}} \cos \left[\left(1 + \frac{BW2}{2} \right) \cdot \frac{\pi}{2} \right] \right\}. \quad (41)$$

Solving $BW2$ as a function of $BW1$ from (39)–(41) we get

$$BW2 = \frac{4}{\pi} \cos^{-1} \left\{ -\sqrt{2} \sin \left[\frac{u_1}{u_2} \left(\cos^{-1} \left(\frac{1}{\sqrt{2}} \cos \left(\left(1 + \frac{BW1}{2} \right) \cdot \frac{\pi}{2} \right) \right) - \frac{\pi}{2} \right) \right] \right\} - 2. \quad (42)$$

Equation (8) gives for the y_a -parameters:

$$y_{a1} = \sqrt{\frac{g_a}{r}} (rg_a - 1) \tan \theta_1 \sin(u_1\theta_1) \cos(u_1\theta_1) \quad (43)$$

$$y_{a2} = \sqrt{\frac{g_a}{r}} (rg_a - 1) \tan \theta_2 \sin(u_2\theta_2) \cos(u_2\theta_2). \quad (44)$$

With (39) the loaded Q -factors Q_{L1} and Q_{L2} are

$$\frac{Q_{L2}}{Q_{L1}} = \frac{y_{a2}}{y_{a1}} = \cot \left[\frac{u_1}{u_2} \cdot \left(\frac{\pi}{2} - \theta_1 \right) \right] \cot \theta_1. \quad (45)$$

B. Circuit a, Two Transformers

From (11) we get the condition

$$\begin{aligned} & \frac{(1 + \sqrt{g_a})^2 \sin^2(u_1\theta_1) \cos^2(u_1\theta_1)}{1 - [\sqrt{g_a} \cos^2(u_1\theta_1) - \sin^2(u_1\theta_1)]^2} \\ & = \frac{(1 + \sqrt{g_a})^2 \sin^2(u_2\theta_2) \cos^2(u_2\theta_2)}{1 - [\sqrt{g_a} \cos^2(u_2\theta_2) - \sin^2(u_2\theta_2)]^2} \end{aligned} \quad (46)$$

which implies

$$u_2\theta_2 = u_1\theta_1 + (u_2 - u_1) \cdot \frac{\pi}{2}. \quad (47)$$

In terms of the bandwidths θ_1 and θ_2 are given by (19)

$$\theta_1 = \cos^{-1} \left\{ \sqrt{\frac{3}{4}} \cos \left[\left(1 + \frac{BW1}{2} \right) \cdot \frac{\pi}{2} \right] \right\} \quad (48)$$

$$\theta_2 = \cos^{-1} \left\{ \sqrt{\frac{3}{4}} \cos \left[\left(1 + \frac{BW2}{2} \right) \cdot \frac{\pi}{2} \right] \right\}. \quad (49)$$

Thus the following relation between the bandwidths:

$$BW2 = \frac{4}{\pi} \cos^{-1} \left\{ -\sqrt{\frac{4}{3}} \sin \left[\frac{u_1}{u_2} \cdot \left(\cos^{-1} \left(\sqrt{\frac{3}{4}} \cos \left(\left(1 + \frac{BW1}{2} \right) \cdot \frac{\pi}{2} \right) \right) - \frac{\pi}{2} \right) \right] \right\} - 2. \quad (50)$$

With the assumption made concerning the frequency independence of y_{T2} , y_{T1} , and g_a it is evident that (12) is satisfied at both the frequencies. The choice of y_{a1} and y_{a2} is (13)

$$y_{a1} = \frac{CD - AB}{B^2 + D^2} \Big|_{u\theta = u_1\theta_1} \cdot \tan \theta_1 \quad (51)$$

$$y_{a2} = \frac{CD - AB}{B^2 + D^2} \Big|_{u\theta = u_2\theta_2} \cdot \tan \theta_2. \quad (52)$$

By insertion of (47) into (51) and (52) we find for the loaded Q -factors,

$$\frac{Q_{L2}}{Q_{L1}} = \frac{y_{a2}}{y_{a1}} = \cot \left[\frac{u_1}{u_2} \left(\frac{\pi}{2} - \theta_1 \right) \right] \cot \theta_1. \quad (53)$$

The conditions derived from (14) are

$$\begin{aligned} & \frac{r_1 - 1}{r_1 + 1} = \\ & \left[\frac{(Dg_a - A - By_{a1} \cot \theta_1)^2 + (C - Bg_a - Dy_{a1} \cot \theta_1)^2}{(Dg_a + A + By_{a1} \cot \theta_1)^2 + (C + Bg_a - Dy_{a1} \cot \theta_1)^2} \right]^{1/2} \Big|_{u\theta = u_1\theta_1} \end{aligned} \quad (54)$$

TABLE I
PARAMETERS FOR THE MATCHING NETWORK ACCORDING TO
FIG 4a, b. (N =NUMBER OF TRANSFORMERS, $R_j=50 \cdot g_a^{-1}$,
 $R_{T1}=50 \cdot y_{T1}^{-1}$ and $R_{T2}=50 \cdot y_{T2}^{-1}$)

N=1		R1=1.20		U1= 3.		U2= 5.	
BW1	BW2	R2	RJ	RT1	RT2	QL1	QL2
0.050	0.030	1.200	2.007	9.145	50.000	12.391	20.654
0.100	0.060	1.200	7.247	17.377	50.000	5.845	9.749
0.150	0.090	1.200	14.028	24.176	50.000	3.572	5.962
0.200	0.120	1.200	20.856	29.479	50.000	2.409	4.026
0.250	0.150	1.200	26.914	33.487	50.000	1.714	2.868
0.300	0.179	1.200	31.945	36.484	50.000	1.261	2.114
0.350	0.209	1.200	35.990	38.724	50.000	0.951	1.597
0.400	0.239	1.200	39.194	40.411	50.000	0.729	1.228
0.450	0.268	1.200	41.719	41.693	50.000	0.566	0.956
0.500	0.297	1.200	43.709	42.676	50.000	0.443	0.751
0.550	0.327	1.200	45.279	43.436	50.000	0.348	0.592
0.600	0.355	1.200	46.519	44.026	50.000	0.273	0.467
0.650	0.384	1.200	47.495	44.485	50.000	0.214	0.366

N=1		R1=1.20		U1= 3.		U2= 7.	
BW1	BW2	R2	RJ	RT1	RT2	QL1	QL2
0.050	0.021	1.200	2.007	9.145	50.000	12.391	28.918
0.100	0.043	1.200	7.247	17.377	50.000	5.845	13.650
0.150	0.064	1.200	14.028	24.176	50.000	3.572	8.351
0.200	0.086	1.200	20.856	29.479	50.000	2.409	5.641
0.250	0.107	1.200	26.914	33.487	50.000	1.714	4.019
0.300	0.128	1.200	31.945	36.484	50.000	1.261	2.965
0.350	0.149	1.200	35.990	38.724	50.000	0.951	2.241
0.400	0.170	1.200	39.194	40.411	50.000	0.729	1.724
0.450	0.191	1.200	41.719	41.693	50.000	0.566	1.344
0.500	0.212	1.200	43.709	42.676	50.000	0.443	1.056
0.550	0.233	1.200	45.279	43.436	50.000	0.348	0.833
0.600	0.253	1.200	46.519	44.026	50.000	0.273	0.657
0.650	0.273	1.200	47.495	44.485	50.000	0.214	0.516
0.700	0.293	1.200	48.259	44.842	50.000	0.165	0.401
0.750	0.313	1.200	48.849	45.115	50.000	0.125	0.305

N=2		R1=1.20		U1= 3.		U2= 5.	
BW1	BW2	R2	RJ	RT1	RT2	QL1	QL2
0.050	0.030	1.200	0.012	0.111	7.156	16.257	27.102
0.100	0.060	1.200	0.193	0.875	14.078	7.947	13.258
0.150	0.090	1.201	0.966	2.854	20.533	5.083	8.490
0.200	0.120	1.201	2.943	6.382	26.306	3.571	5.975
0.250	0.150	1.201	6.669	11.404	31.226	2.600	4.361
0.300	0.180	1.202	12.218	17.402	35.205	1.913	3.217
0.350	0.210	1.202	19.002	23.594	38.276	1.404	2.369
0.400	0.239	1.202	26.063	29.285	40.567	1.024	1.734
0.450	0.269	1.202	32.518	34.041	42.218	0.743	1.264
0.500	0.299	1.201	37.940	37.830	43.420	0.532	0.909
0.550	0.328	1.201	42.227	40.659	44.252	0.376	0.646

N=2		R1=1.20		U1= 3.		U2= 7.	
BW1	BW2	R2	RJ	RT1	RT2	QL1	QL2
0.050	0.021	1.200	0.012	0.111	7.156	16.257	37.945
0.100	0.043	1.200	0.193	0.875	14.078	7.947	18.566
0.150	0.064	1.201	0.966	2.854	20.533	5.083	11.893
0.200	0.086	1.201	2.943	6.382	26.306	3.571	8.374
0.250	0.107	1.201	6.669	11.404	31.226	2.600	6.116
0.300	0.128	1.202	12.218	17.402	35.205	1.913	4.515
0.350	0.150	1.202	19.002	23.594	38.276	1.404	3.328
0.400	0.171	1.202	26.063	29.285	40.567	1.024	2.439
0.450	0.192	1.202	32.518	34.041	42.218	0.743	1.779
0.500	0.213	1.201	37.940	37.830	43.420	0.532	1.181
0.550	0.234	1.201	42.227	40.659	44.252	0.376	0.913

$$\frac{r_2 - 1}{r_2 + 1} = \frac{\left[(Dg_a - A - By_{a2} \cot \theta_2)^2 + (C - Bg_a - Dy_{a2} \cot \theta_2)^2 \right]^{1/2}}{\left[(Dg_a + A + By_{a2} \cot \theta_2)^2 + (C + Bg_a - Dy_{a2} \cot \theta_2)^2 \right]^{1/2}} \quad (55)$$

TABLE II
PARAMETERS FOR THE MATCHING NETWORK ACCORDING TO
FIG 4c, d. (N =NUMBER OF TRANSFORMERS, $R_j=50 \cdot g_a^{-1}$,
 $R_{T1}=50 \cdot y_{T1}^{-1}$ and $R_{T2}=50 \cdot y_{T2}^{-1}$)

N=1		R=1.20		U1= 3.		U2= 5.	
BW1	BW2	RJ	RT1	RT2	QL1	QL2	
0.050	0.030	2.003	9.136	50.000	12.379	20.631	
0.100	0.060	7.194	17.314	50.000	5.825	9.709	
0.150	0.090	13.828	24.004	50.000	3.551	5.918	
0.200	0.120	20.403	29.157	50.000	2.392	3.986	
0.250	0.150	26.133	32.998	50.000	1.704	2.840	
0.300	0.180	30.795	35.821	50.000	1.263	2.105	
0.350	0.210	34.447	37.885	50.000	0.967	1.612	
0.400	0.240	37.241	39.391	50.000	0.763	1.222	
0.450	0.270	39.333	40.483	50.000	0.621	1.035	
0.500	0.300	40.854	41.259	50.000	0.522	0.870	
0.550	0.330	41.905	41.786	50.000	0.456	0.761	
0.600	0.360	42.552	42.107	50.000	0.417	0.695	
0.650	0.390	42.838	42.248	50.000	0.399	0.666	

N=1		R=1.20		U1= 3.		U2= 7.	
BW1	BW2	RJ	RT1	RT2	QL1	QL2	
0.050	0.021	2.003	9.136	50.000	12.379	28.884	
0.100	0.043	7.194	17.314	50.000	5.825	13.593	
0.150	0.064	13.828	24.004	50.000	3.551	8.285	
0.200	0.086	20.403	29.157	50.000	2.392	5.581	
0.250	0.107	26.133	32.998	50.000	1.704	3.976	
0.300	0.129	30.795	35.821	50.000	1.263	2.948	
0.350	0.150	34.447	37.885	50.000	0.967	2.257	
0.400	0.171	37.241	39.391	50.000	0.763	1.781	
0.450	0.193	39.333	40.483	50.000	0.621	1.449	
0.500	0.214	40.854	41.259	50.000	0.522	1.219	
0.550	0.236	41.905	41.786	50.000	0.456	1.065	
0.600	0.257	42.552	42.107	50.000	0.417	0.972	
0.650	0.279	42.838	42.248	50.000	0.399	0.932	

N=2		R=1.20		U1= 3.		U2= 7.	
BW1	BW2	RJ	RT1	RT2	QL1	QL2	
0.050	0.021	0.012	0.111	7.142	16.240	37.894	
0.100	0.043	0.189	0.858	13.970	7.915	18.448	
0.150	0.064	0.917	2.738	20.213	5.039	8.398	
0.200	0.086	2.705	5.971	25.670	3.522	5.870	
0.250	0.107	5.939	10.422	30.241	2.553	4.255	
0.300	0.129	10.624	15.638	33.924	1.872	3.120	
0.350	0.150	16.293	20.996	36.784	1.375	2.291	
0.400	0.171	22.217	25.964	30.954	1.009	1.681	
0.450	0.193	27.668	30.157	40.545	0.745	1.241	
0.500	0.214	32.191	33.443	41.686	0.559	0.932	
0.550	0.236	35.543	35.787	42.453	0.436	1.017	

Regarding the fact that $BW1$ and $BW2$ are related according to (50), only one of the parameters r_1 and r_2 can be chosen independently.

C. Circuit b

Circuit *b* is characterized by having all the distributed lines equally long. The periodic properties of such networks are an indication that simultaneous Chebyshev response around f_1 and f_2 could be obtained, if all the immittance parameters of the network are kept constant. The truth of this supposition can be verified by application of the design equations (21)–(34) at f_1 and f_2 as follows.

For any one of (24), (33), or (34) we get

$$\theta_1 = \cos^{-1} \left\{ A \cos \left[\left(1 + \frac{u_1 \cdot BW1}{2} \right) \cdot \frac{\pi}{2} \right] \right\} \quad (58)$$

$$\theta_2 = \cos^{-1} \left\{ A \cos \left[\left(1 + \frac{u_2 \cdot BW2}{2} \right) \cdot \frac{\pi}{2} \right] \right\} \quad (59)$$

where $A = 1/\sqrt{2}$, $\sqrt{\frac{3}{4}}$, or $\frac{1}{2}$, respectively.

By choosing $BW1$ and $BW2$ inversely proportional to f_1 and f_2 and thus to u_1 and u_2 we can find a basic bandwidth, BW , such that

$$BW1 = \frac{BW}{u_1} \quad (60)$$

$$BW2 = \frac{BW}{u_2}. \quad (61)$$

Insertion of (60) and (61) into (58) and (59) gives

$$\theta_1 = \theta_2. \quad (62)$$

Thus all the design equations are made equal for $f=f_1$ and $f=f_2$, implying identical networks. Therefore, we have

$$y_{b2} = y_{b1} \quad (63)$$

$$r_2 = r_1. \quad (64)$$

From (60) and (61) we get

$$\frac{BW2}{BW1} = \frac{u_1}{u_2}. \quad (65)$$

The Q -factors are related as

$$\frac{Q_{L2}}{Q_{L1}} = \frac{f_2}{f_1} \cdot \frac{\frac{d}{df} [-y_{b2} \cot(u_2 \theta)]}{\frac{d}{df} [-y_{b1} \cot(u_1 \theta)]} \Bigg|_{\theta=\frac{\pi}{2}} \quad (66)$$

which, considering (63), gives

$$\frac{Q_{L2}}{Q_{L1}} = \frac{u_2}{u_1} = \frac{f_2}{f_1}. \quad (67)$$

The preceding equations have been used in a computer program giving design and performance data for the circuits. Some results are given in Tables I and II.

In this section we have found the conditions for Chebyshev response at two frequencies. The results can, however, be applied to an arbitrary number of frequencies by demanding the conditions, as given above, to be fulfilled for every pair of frequencies.

V. REALIZATION OF MULTIPLE FREQUENCY CIRCULATOR JUNCTIONS FOR CHEBYSHEV MATCHING

In Section IV design equations were obtained specifying the properties of the equivalent networks for circulator junctions in order to enable simultaneous Chebyshev response within multiple frequency bands. More specifically: If, for one of the given types of networks, we choose the ratio of resonance frequencies together with the maximum VSWR and bandwidth for matching around one, given resonance frequency, then the network is completely determined, as exemplified in Tables I and II. Thus at each resonance the shunt conductance, the characteristic admittance and the length of the short-circuited stub is uniquely specified. If, instead of the equivalent network, we insert the circulator junction, then the equivalent admittance of the junction, $y_{eq} = g_{eq} + jb_{eq}$, must satisfy three corresponding conditions at each resonance frequency f_r .

Network *a*:

$$b_{eq}(f_r) = 0 \quad (68)$$

$$g_{eq}(f_r) = g_a(f_r) \quad (69)$$

$$\frac{db_{eq}(f)}{df} \Bigg|_{f_r} = \frac{\pi \cdot y_a(f_r)}{2f_r}. \quad (70)$$

Network *b*:

$$b_{eq}(f_r) = 0 \quad (71)$$

$$g_{eq}(f_r) = g_b(f_r) \quad (72)$$

$$\frac{db_{eq}(f)}{df} \Bigg|_{f_r} = \frac{u \cdot \pi y_b(f_r)}{2f_r}. \quad (73)$$

Equations (70) and (73) are more conveniently expressed in terms of the Q -factors, Q_a and Q_b , of the junction

$$Q_a(f_r) = \frac{\pi y_a(f_r)}{4g_a(f_r)} \quad (74)$$

$$Q_b(f_r) = u \cdot \frac{\pi y_b(f_r)}{4g_b(f_r)} \quad (75)$$

where, related to y_{eq} , the Q -factor is given by

$$Q(f_r) = \frac{f_r}{2g_{eq}(f_r)} \cdot \frac{db_{eq}(f)}{df} \Bigg|_{f_r}. \quad (76)$$

In order to be able to match a junction for Chebyshev response around N circulation frequencies, we thus have $3N$ conditions on y_{eq} to satisfy.

By solving the boundary value problem associated with a junction the electric properties can be expressed in terms of an infinite series of modes. It is found that a good approximation is obtained by taking into account only a few modes giving the main part of the contribution [10]. Though computer-aided synthesis of junctions satisfying the conditions for single frequency band operation is readily made, it is a very time consuming task to find junctions satisfying also the additional conditions demanded for multi-frequency operation. However, by making the approximation of taking into account only the dominant mode related to each circulation, analytic solutions for the general prob-

lem of multifrequency circulation have been found [9]. Here we will use these single mode solutions to derive a theorem concerning the possibility of making Chebyshev matched double frequency circulators. In this derivation is assumed that the material is lossless.

The conditions for circulation used in [9] are that the equivalent wave impedance should be real and equal to the intrinsic wave impedance of the surrounding medium. Assuming a proportionality between the equivalent wave impedance and the equivalent impedance and frequency independent intrinsic impedance of the surrounding medium, these conditions could be written

$$b_{eq} = 0 \quad (77)$$

$$g_{eq} = \alpha \quad (78)$$

where α is an admittance, independent of frequency. By choosing the proper thickness of the junction, α could be made equal to the prescribed g_a or g_b . Thus the solutions for multifrequency circulation presented in [9], satisfy two of the three conditions for Chebyshev matched circulation at every circulation frequency according to (68) and (69), or (71) and (72).

The third condition for Chebyshev matched circulation concerns the Q -factor. In [9] an accurate expression for the Q -factor was derived:

$$Q_L = \frac{X^2 - n^2}{2\sqrt{3}n} \left| \frac{\mu}{\kappa} \right| \left\{ 1 + \frac{f_c^2 f_m (f_m + f_0)}{[(f_m + f_0)^2 - f_c^2][f_0(f_m + f_0) - f_c^2]} \right\} \quad (79)$$

where

X	the corresponding solution of $J'_n(X) = 0$,
$J_n(X)$	Bessel function of the first kind,
n	order of the Bessel function,
μ, κ	elements for the Polder tensor of the ferrite.
$f_m =$	$\frac{\gamma M_s}{2\pi}$
$f_0 =$	$\frac{\gamma H_i}{2\pi}$
f_c	circulation frequency,
γ	gyromagnetic ratio,
M_s	saturation magnetization,
H_i	internal biasing magnetic field intensity.

The expressions for μ and κ used in [9] are

$$\mu = 1 + \frac{f_m f_0}{f_0^2 - f^2} \quad (80)$$

$$\kappa = \frac{f_m f}{f_0^2 - f^2} \quad (81)$$

where f is the signal frequency.

The conditions for circulation at two frequencies, f_1 and f_2 , with a surrounding material having frequency independent properties are [9]

$$f_0 = \sqrt{\frac{1 - \alpha_{1,2} \beta_{1,2}}{f_2^2 - \alpha_{1,2} \beta_{1,2} f_1^2}} \cdot \frac{(\beta_{1,2} - \alpha_{1,2}) f_1^2 f_2^2}{\beta_{1,2} f_2^2 - \alpha_{1,2} f_1^2} \quad (82)$$

$$f_m = \frac{\beta_{1,2} [f_2^4 - (1 + \alpha_{1,2}^2) f_1^2 f_2^2 + \alpha_{1,2} f_1^4]}{\sqrt{(1 - \alpha_{1,2} \beta_{1,2})(f_2^2 - \alpha_{1,2} \beta_{1,2} f_1^2)} (\beta_{1,2} f_2^2 - \alpha_{1,2} f_1^2)} \quad (83)$$

where

$$\alpha_{1,2} = \frac{X_2}{X_1} \quad (84)$$

$$\beta_{1,2} = S_{1,2} \frac{X_1 n_2^3 \sin^2(n_1 \Psi) \sin(2n_1 \pi/3)}{X_2 n_1^3 \sin^2(n_2 \Psi) \sin(2n_2 \pi/3)} \quad (85)$$

where 2Ψ = coupling angle for the transmission lines connecting to the junction and $S = +1$ or -1 indicates the relative circulation directions. The indexes 1 and 2 are related to the circulation frequencies f_1 and f_2 , respectively.

By choosing the same dominant circulation mode above and below ferromagnetic resonance (84) and (85) are simplified according to

$$\alpha_{1,2} = 1 \quad (86)$$

$$\beta_{1,2} = -1 \quad (87)$$

the minus sign in (87) being due to the different circulation directions. By insertion of (80)–(83) and (86)–(87) into (79), and evaluation of the Q -factors, Q_{L1} and Q_{L2} , at f_1 and f_2 we get, after simplification,

$$Q_{L1} = \sqrt{\frac{2}{3}} \cdot \frac{X^2 - n^2}{n} \cdot \frac{f_1 \sqrt{f_2^2 + f_1^2}}{f_2^2 - f_1^2} \quad (88)$$

$$Q_{L2} = \sqrt{\frac{2}{3}} \cdot \frac{X^2 - n^2}{n} \cdot \frac{f_2 \sqrt{f_2^2 + f_1^2}}{f_2^2 - f_1^2} \quad (89)$$

thus we have

$$\frac{Q_{L2}}{Q_{L1}} = \frac{f_2}{f_1} \quad (90)$$

By looking at the conditions for Chebyshev response at two frequencies derived in Section IV we find that exactly the same relation is required according to (67), when we use an equivalent network of type *b*. Thereby the following theorem has been established: If a junction is constructed for double frequency operation with the same circulation mode below and above resonance, and if it is matched for Chebyshev response around one of the circulation frequencies with a matching network of proper length, then it is also matched for Chebyshev response around the other circulation frequency.

The ratio Q_{L2}/Q_{L1} has been computed for several different lower order mode combinations using single mode description and compared with f_2/f_1 . Equality has been found for no other mode combinations than the ones treated above.

VI. DESIGN EXAMPLES

As shown in Section V the use of the same dominant mode below and above resonance is advantageous to obtain Chebyshev matched double frequency circulators. In Fig. 5 the response of a single transformer matched junc-

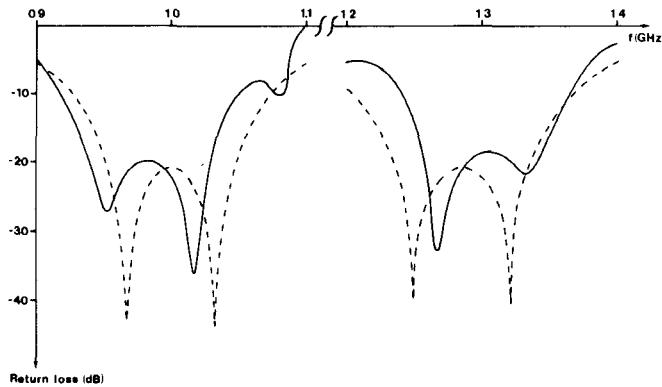


Fig. 5. Single transformer matched double frequency circulator. Return loss (—) as given by single mode analysis compared with ideal Chebyshev response (---).

tion working in the (1,1)-mode is shown. If we choose $f_2/f_1=9/7$, the Q -factors of the junction are 4.87 and 6.26. By the aid of the matching network the nominal bandwidths corresponding to VSWR equal to 1.2 are increased from 3.8 and 2.9 to 9.9 and 7.7 percent, respectively. Due to the small difference between the input impedance characteristics of the junction and its equivalent circuit the bandwidths obtained with the junction are reduced to 9.0 and 7.5 percent and the VSWR values are increased to 1.23 and 1.27.

Simultaneous Chebyshev matching at more than one frequency is normally impossible to obtain when different dominant modes are used. A considerable improvement of the nominal bandwidth can, however, often be obtained by using the matching networks described above. The synthesis procedure is as follows.

- 1) Choose dominant mode combination f_2/f_1 , number of transformers and equivalent network of the junction.
- 2) Design the junction, compute the corresponding Q_{L1} and Q_{L2} for the junction.
- 3) Compute the equivalent network shunt admittances, g_1 and g_2 , corresponding to Q_{L1} and Q_{L2} , required for Chebyshev matching with prescribed ripple.
- 4) Choose as a compromise the shunt admittance $g=\sqrt{g_1 \cdot g_2}$.

- 5) Design the matching network corresponding to g .

The use of the procedure above will now be illustrated by an example following the Steps 1-5.

- 1) The mode combination is the (1,1)-mode above and the (2,1)-mode below resonance. $f_1=1$ GHz and $f_2=2.33$ GHz ($f_2/f_1=7/3$). The matching network consists of two transformers. The equivalent network of Fig. 1(b) is used.
- 2) By the use of [9] we get $Q_{L1}=2.34$ and $Q_{L2}=4.37$.
- 3) The corresponding shunt admittances for Chebyshev response, $VSWR=1.2$ is given by Table II, $N=2$, $R=1.2$, $U1=3$, and $U2=7$. The result is $g_1=0.134$ and $g_2=0.094$.
- 4) Choose the compromise $g=\sqrt{0.134 \cdot 0.094}=0.113$. ($R_j=1/0.113=8.89 \Omega$).
- 5) The compromised matching network corresponding to $R_j=8.89 \Omega$, can now be found by interpolation in Table II, $N=2$, $R=1.2$, $U1=3$, and $U2=7$. This gives a matching network composed of two transformers with characteristic resistances $RT1=13.7 \Omega$ and $RT2=36.6 \Omega$, respectively.

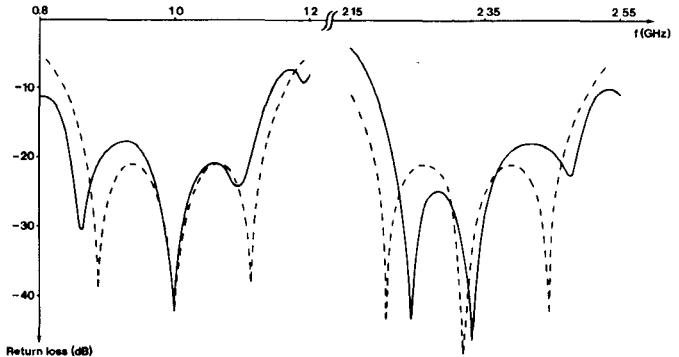


Fig. 6. Double transformer matched double frequency circulator. Return loss (—) as given by single mode analysis compared with ideal Chebyshev response (---).

The results of an analysis of the matched circulator is shown in Fig. 6. The derived bandwidths are 26.0 and 12.0 percent compared with 26.6 and 12.9 percent, respectively, for the hypothetical case with perfect Chebyshev matching around both the frequencies. The maximal VSWR has also been changed from nominally 1.20 to 1.29. The difference between the responses of the matched junction and the corresponding Chebyshev responses is in this case due to: 1) the difference between the properties of the junction and its equivalent networks around the two center frequencies, and 2) the fact that the junction does not satisfy the conditions for simultaneous Chebyshev matching.

The direction of circulation for any circulation mode is given by the sign of A [9] where

$$A = \frac{\sin(n \cdot 2\pi/3)}{\kappa/\mu}, \quad n \neq 0, 3, 6, \dots \quad (91)$$

Since the signs of the splitting, κ/μ , are opposite for above and below resonance operation the circulation directions are opposite for the (1,1), (1,1)-circulator. The (1,1), (2,1)-circulator has equal circulation directions since, in addition, the signs of the sinus factors are opposite for the (1,1), and the (2,1)-modes.

VII. CONCLUSIONS

Two equivalent networks for circulator junction ports are described. Both are shown to be useful for simultaneous multiple frequency matching with Chebyshev response by transformers an odd multiple of a quarter-wavelength long at the center frequencies. The conditions to be fulfilled by the circulator junction to enable multiple frequency Chebyshev matching are stated. By using the simplified theoretical description of the junction in assuming single mode operation it is shown that circulators operating in the same mode below and above resonance can be matched for Chebyshev response. For junctions, where Chebyshev matching is impossible, a procedure for broad-band double frequency matching is proposed, giving in some cases, bandwidths only slightly smaller than those of perfect Chebyshev response. Examples of matched junctions are given indicating that bandwidths in the order of 25 and 10 percent, respectively, should be possible to obtain for double frequency circulators.

The design accuracy of matched multiple frequency circulators can be enhanced by including a multimode description of the junction. The design based on a single mode description, which is readily found based on the theory given above and in [9], can thereby serve as a good starting point.

Due to the fact that the matching transformers considered in this paper may have a length of several quarter wavelengths a higher insertion loss can be expected than for conventional circulators. It is therefore a subject for further study to investigate the possibility of matching stripline circulators over multiple frequency bands by using electrically shorter networks. The magnetic losses can be kept small if the operation frequencies are chosen far enough from the low field loss and the material resonance regions.

Though the multiple frequency band matching technique discussed in this paper has been applied only to stripline junction circulators, it is evident that the results may be applicable also to many other components. A condition is that the components must have an equivalent admittance which over repeated frequency bands can be approximated by that of one of the equivalent networks presented in Section II. Examples of candidate components are other types of junction circulators, loaded resonators, and certain multiport components [11], [12].

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